

CATS-2025 Lightning Talks

April 6, 2025

Speaker: Maria Akter (Alabama)

Title: On $\mathfrak{m}$ -adic continuity of F-splitting Ratio

Abstract. Let $(R, \mathfrak{m}, k)$ be a local F-finite ring of prime characteristics $p > 0$. For each $e \in \mathbb{N}$, let $F_*^e R$ be the finitely generated R -module obtained via the restriction of scalars under the e th iteration of Frobenius map. If $e \in \mathbb{N}$, consider the e th Frobenius splitting number of R , denoted by a_e , defined by $F_*^e R \cong \oplus R^{a_e} \oplus M_e$, where M_e has no free R -summand. The Frobenius splitting ratio of R is the asymptotic rate of the number a_e grow as $e \rightarrow \infty$. If $(R, \mathfrak{m}, k)$ be a local F-finite $\mathbb{Q}$ -Gorenstein Cohen-Macaulay ring of prime characteristic $p > 0$, $0 \neq f \in \mathfrak{m}$ so that $R/(f)$ is (G_1) and (S_2) and $\Delta \geq 0$ be an effective $\mathbb{Q}$ -divisor of R such that $K_R + \Delta$ is $\mathbb{Q}$ -Cartier and $(p^e - 1)\Delta$ is integral for all sufficiently large $e \gg 0$, then the Frobenius splitting numbers of the pair $(R/(f), \Delta|f)$ can be measured through compatible Frobenius splitting in (R, Δ) via techniques of Inversion of Adjunction of F-purity. The talk will involve this technique which allows comparisons between the Frobenius splitting ratio of the pair $(R/(f), \Delta|f)$ and $(R/(f + \varepsilon), \Delta|(f + \varepsilon))$ for $\varepsilon \in \mathfrak{m}^N$ and $N \gg 0$.

Speaker: Benjamin Baily (Michigan)

Title: Classification of Minimal Singularity Thresholds

Abstract: Let X be a smooth complex variety and Y a proper closed subscheme of X . The log canonical threshold of the pair (X, Y) is a positive rational number which measures the singularities of Y , and the study of this quantity is an important topic in birational geometry with connections to the minimal model program. In positive characteristic, one instead typically studies the F-pure threshold of a pair.

In 2014, Demailly and Pham defined an invariant of a pair $(\text{Spec } R, \text{Spec } R/I)$ in terms of the Segre numbers of I and showed that this invariant is a sharp lower bound on the log canonical threshold. We show that in positive characteristic, the same invariant is a lower bound on the F-pure threshold. Further, when R is a polynomial ring over an algebraically-closed field of characteristic 0 (resp. $p > 0$), we classify all homogeneous ideals I for which the log canonical threshold (resp. F-pure threshold) of $(\text{Spec } R, \text{Spec } R/I)$ is equal to Demailly and Pham's invariant.

Speaker: Sudip Das (Arizona State)

Title: Asymptotic colengths for families of ideals: an analytic approach

Abstract: This work focuses on the existence of asymptotic colengths for families of $\mathfrak{m}$ -primary ideals in a Noetherian local ring $(R, \mathfrak{m})$. In any characteristic, we generalize graded families to weakly graded families of ideals, and in prime characteristic, we explore various families such as weakly p -families and weakly inverse p -families. The main contribution of this paper is providing a unified analytic method to prove the existence of limits. Additionally, we establish Brunn-Minkowski type inequalities, positivity results, and volume = multiplicity formulas for these families of ideals.

Speaker: Sean Grate (Auburn)

Title: Combinatorial bounds on the Castelnuovo-Mumford regularity of toric surfaces

Abstract: In 1996, L'vovsky showed the Castelnuovo-Mumford regularity of the coordinate ring of a monomial curve is bounded by the sum of its semigroup's two largest gaps. We explore analogous results for toric surfaces embedded by incomplete linear systems, and show that for certain classes the regularity is controlled by the combinatorics of the associated semigroup.

Speaker: Dipendranath Mahato (Tulane)

Title: Interpolation in Projective Spaces

Abstract: In N -dimensional Projective Space, the main problem of interpolation is about finding the minimal degree of homogeneous polynomial that vanishes on a finite set of points with a given set of multiplicities. To deal with such a problem G. V. Chudnovsky and J.P. Demailly provided some conjectural bounds to the minimal degree, which I will discuss in the lightning session.

Speaker: Jiamin Pan (Georgia State)

Title: Computing Boolean Functions from Polynomial Ideals with Prescribed Zeros

Abstract: We discuss an algebra-related problem on the synthesis of Boolean functions from polynomial ideals, with applications in computer engineering, explain the role of von Neumann regular rings in this setting, and present a theoretical solution.

Speaker: Shiji Lyu (Illinois at Chicago)

Title: Sagata rings

Abstract: A Nagata ring is a Noetherian ring R so that every finite R -algebra B that is an integral domain has finite normalization. Parallel to Nagata rings, I define a Sagata ring to be a Noetherian ring R so that every finite R -algebra B that is an integral domain has a finite (S_2) -ification. I will discuss analogous results and new difficulties/phenomena.

Speaker: Songhao Zhu (Georgia Tech)

Title: There are enough spherical highest weights

Abstract: We show that, even for Lie superalgebras, there are enough, meaning Zariski dense, highest weights for irreducible, finite-dimensional, and spherical weights for a (g, k) -pair. A commutative diagram will also be presented.

Speaker: Phan Vinh (Tulane)

Title: Limits of length function of multi- p -family of ideals

Abstract: Many results showed the existence of the mixed multiplicities of a multi-graded family of ideals. In this study, we observed that one of the main tools in proving the existence of mixed multiplicity in the graded case still holds for the case of p -families. We proved the asymptotic relationship between the limit of the normalized length function of a multi- p -family of $\mathfrak{m}_R$ -primary ideals and that of its shifted family under a linear growth assumption in a local domain of characteristic p . In this talk, we will discuss the proof of the theorem. This is joint work with Thai Thanh Nguyen.
